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A Historical Perspective on Unconfined Seepage in Geotechnical Analysis: Correcting a Common Fallacy

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Abstract
It is commonly assumed in seepage studies for geotechnical problems that the phreatic surface (the line of zero pore water pressure) represents the upper flow line and is the boundary of the seepage zone. Seepage is assumed to occur below the phreatic surface but not above it (this is termed the phreatic assumption). A brief historical review is given of the development of seepage theory and how this assumption became established. It is explained that this assumption is only true if the soil involved is coarse grained when the majority of the pore water above the phreatic surface will drain out of it under gravity forces. The assumption is not valid for fine-grained soils such as clay, in which case water is held in the soil pores by capillary forces for many metres above the phreatic surface and seepage occurs above the phreatic surface according to the same laws as below it. Examples are given of correct flow nets for unconfined seepage in clay compared with those routinely adopted in soil mechanics.

Keywords
Dams, barrages & reservoirs; earth dams; Groundwater.

Notation

A : Area of flow
D_{10}: 10% particle size of soil sample
e: Void ratio
g: Acceleration due to gravity
h: Total hydraulic head
h_e: Elevation head
h_p: Pressure head
Δh: Difference in hydraulic head
i : Hydraulic gradient
K : Permeability (hydraulic conductivity, coefficient of permeability)
K_r : Relative permeability (ratio of permeability at a given suction to the permeability at zero suction)
K_s : Permeability at a certain suction value
K_0 : Permeability at zero suction
k: Intrinsic permeability
l: Length of flow path
N_e: Number of head drops in a flow net
n: Porosity
p: Water pressure
Q: Flow rate
u: Pore water pressure
u_a: Air entry pressure
v: Velocity
v_D: Darcy (or discharge) velocity
v_T: True (or travel time) velocity
x, y: Coordinates in two dimensions
ρ : Density of permeating fluid
μ : Dynamic viscosity of permeating fluid
γ_w : Unit weight of water.

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Introduction

Groundwater is fundamental to all aspects of geotechnical engineering. As Professor William Powrie said in his review of groundwater papers from the first 60 years of the *Géotechnique* journal 'Without groundwater, geotechnical engineering would be much simpler − but also less of a challenge and much less interesting' (Powrie, 2008).

Accordingly, the analysis of groundwater seepage problems is a key part of the academic training and professional development of many geotechnical engineers and engineering geologists. However, given the wide range of soil types (where the permeability may vary by 10 to 12 orders of magnitude between a very low permeability clay and a very coarse gravel) and different seepage problems, in the authors' experience seepage concepts and models can sometimes be applied inappropriately.

A key application of groundwater seepage analysis is the large scale seepage through and beneath a dam or earth embankment, where the structure is used to retain water or wet mine tailings. The commonly applied solutions to these problems assume that the phreatic surface (the line of zero pore water pressure) is the upper flow line and is the boundary of the seepage zone. However, this assumption (here termed the phreatic assumption) is only true if the soil involved is coarse grained when the majority of the pore water will drain out of it under gravity forces. The phreatic assumption cannot be valid for clay, in which case water is held in the soil pores by capillary forces for many metres above the phreatic surface. We know well that water retaining embankments almost invariably are made of clay or use clay for their water retaining component, so it is surprising that the phreatic assumption has survived for so long in this context.

This paper presents a historical review of the development of seepage theory and how the phreatic assumption became established. It is emphasised that in clays and other fine-grained soils seepage occurs above the phreatic surface according to the same laws as below it. Examples are given of correct flow nets for unconfined seepage in clay compared with those routinely adopted in geotechnical analyses.

Darcy's Law and Classical Seepage Theory

Precise analysis of the flow of groundwater through soil is probably one of the more complex mathematical problems faced in routine geotechnical analysis. But for the vast majority of geotechnical problems, groundwater flow can be described by the deceptively simple Darcy's law.

In the mid 19th century, Henri Darcy made an extensive study of the problems of obtaining an adequate supply of potable water for the town of Dijon (Freeze, 1994). His extensive report *Les Fontaines Publique de la Ville de Dijon* (Darcy, 1856) analysed the available sources of water from both rivers and wells—some of them artesian—and how to economically harness all these for optimum usage. However, this work is largely remembered for Darcy's law postulating how to determine the 'permeability' of a column of sand of selected grading, knowing the rate of water flow through it. Darcy's apparatus is shown in Figure 1.

Permeability (more correctly known as coefficient of permeability) is a key geotechnical parameter relating to groundwater flow through soil and rock. Essentially, permeability is a measure of the ease or otherwise with which a fluid passes through a porous medium. A complication is that the flow-head relationship is controlled not only by the nature of the porous media, but also by the properties of the permeating fluid. Therefore, the permeability of a soil or rock to water is different from the permeability to another fluid, such as air or oil (the permeability parameter, independent of the fluid, is known as the intrinsic permeability). Seepage studies and hydrogeology references highlight this by using the term 'hydraulic conductivity' to show that the permeability parameter used in this field is specific to water.

Civil and geotechnical engineers deal almost exclusively in the flow of water through soils and rocks and use the term "coefficient of permeability," instead of hydraulic conductivity, which is given the symbol *K*. For convenience, *K* is generally referred to simply as 'permeability' and this terminology will be used in the current paper.

Darcy's law uses permeability to relate flow rate *Q* through a soil zone of cross sectional area *A*, due to an hydraulic gradient *i* (the hydraulic gradient is created by the existence of a head difference Δh). *Q* is initially expressed in terms of the intrinsic permeability *k* and the properties of the permeating fluid (density ρ and dynamic viscosity μ):

$$Q = -\left(\frac{k\rho g}{\mu}\right) iA \tag{1}$$

if water is the permeating fluid, this becomes the commonly used form of Darcy's law:

$$Q = -KiA \tag{2}$$

where *K* is the coefficient of permeability ("permeability" or hydraulic conductivity). The negative term in equations 1 and 2 is necessary because flow occurs down the hydraulic gradient – i.e. from high head to low head. Darcy's law is typically illustrated by a laboratory seepage experiment (Figure 2), with hydraulic gradient expressed in terms of Δh and flow path length *l* to give

$$Q = -K\frac{\Delta h}{l}A \tag{3}$$

Darcy's law is (along with Terzaghi's effective stress equations, (Terzaghi, 1925)) a key underpinning of many forms of geotechnical analysis. It is often forgotten that Darcy's law is not a theoretical development, but is an empirical rule identified by Darcy from the results of his original experiments.

Darcy's law is predicated on laminar flow (termed Darcian flow) where, for a given geometry, *Q* and Δh have a linearly proportional relationship. At higher Reynolds Numbers (essentially at higher hydraulic gradients), flow becomes turbulent or non-Darcian and there is no longer a linear relationship between *Q* and Δh. It is generally accepted that, for the range of hydraulic gradients encountered in most geotechnical problems, Darcian flow will dominate in groundwater seepage. However, in rock where more open fractures are present, and if hydraulic gradients are high, non-Darcian (turbulent) flow will occur, and *Q* will increase under-proportionally with Δh as energy is lost to turbulence. An example of a case where non-Darcian flow may occur is packer permeability tests in rock with very open fractures (Preene, 2018). In most other geotechnical problems, hydraulic gradients are low enough to ensure Darcian flow conditions prevail. The discussion in the remainder of this paper is based on the presumption of Darcian flow, which is the underlying assumption in most methods of geotechnical analysis and modelling.

Figure 2 and equation 3 show that Darcy's law relates groundwater flow to "head", which is best defined by its physical reality, as follows:

Head is the level to which water will rise in an open standpipe with its lower end at the point under consideration (e.g. Strack, 1989).

Understanding and solving seepage problems is greatly helped by keeping this definition firmly in mind.

Head is also more correctly known as total hydraulic head or just hydraulic head. Figure 3 shows two points in the ground having the same "head" (or total hydraulic head) at a given point. This is the sum of the “pressure head” h_p and the “elevation head” h_e. The elevation head is the height of the measuring point above an arbitrary datum, and the pressure head is the pore water pressure *u*, expressed as metres of head of water. Lines of equal head are known as equipotential lines, and are widely used in analysing seepage situations.

From Figure 3 we can write the well known expression for total head

$$h = h_e + h_p = h_e + \frac{u}{\gamma_w} \qquad (4)$$

where u = pore pressure

and γ_w = unit weight of water

In connection with equation 4 reference should also be made to the work of Bernoulli (1738), whose specialty was the field of fluid mechanics, mainly with reference to flow in pipes. HIs well known equation, derived by considering energy concepts, is as follows:

$$h = h_p + \frac{p}{\gamma_w} + \frac{v^2}{2g} \qquad (5)$$

where p = water pressure

v = velocity

g = gravity.

Compared to flow in pipes, the velocity of seeping water in the ground is extremely small and the velocity term in equation 5 can be neglected. Equation 5 then becomes identical with equation 4 (where the water pressure *p* is taken to be the pore water pressure *u*).

Head (or total hydraulic head) is supremely important because it controls groundwater flow. Water will flow from high total hydraulic head to low total hydraulic head. This important concept is often mis-understood, and means that water does not necessarily flow from high pressure to low pressure or from high elevations to low elevations—it will only flow in response to differences in head, not pressure or elevation considered in isolation. This is illustrated in Figure 4, which shows three situations where water flows from point A to point B under the same hydraulic gradient $i = \Delta h/l$. In Figure 4(a) the pressure at A is greater than at B and seepage is horizontal. In Figure 4(b) the pressure at A is less than at B but seepage still takes place from A to B. In Figure 4(c) the pressure at A is greater than at B and flow takes place "up-hill" from A to B. It is clear therefore that it is a difference in head that governs seepage behaviour.

The equation describing Darcy’s Law, as noted later, appears the same as the laws governing heat and electricity flow in conducting mediums, usually metals. However, there is one important difference. Unlike the materials that heat and electricity flow through, soil is not a single uniform material; rather it consists of solid particles and void space; the soil particles are dead space fpr flow and water only flows through the void space. Therefore, the velocity of the seeping water in the soil pores is greater than that implied by Darcy’s Law. The space the water travels through is defined by the porosity of the soil (the ratio of pore space to total volume of the soil), and thus the velocity is given by:

$$v_T = v_D/n$$

where v_D = Darcy (or discharge) velocity

v_T = True (or travel time) velocity
n= porosity

Historically, this issue has been of little significance as geotechnical engineers have been concerned almost exclusively with flow rates. However, the greatly increased concern with environmental effects in recent years has included contamination of groundwater, and thus on the rate at which contaminants are carried by advection with moving water. The porosity is invariably less than unity so the true velocity is always greater than the Darcy velocity.

Groundwater Flow in the Partially Saturated or Unsaturated Zone

The terms unsaturated and partially saturated are commonly used interchangeably in the literature; both indicate that the soil pores are not completely filled with water, but rather contain a mixture of water and air or water vapour.

The phreatic surface is defined as the line of zero pore water pressure (i.e. u = 0). In many geotechnical analyses it is assumed that the phreatic surface represents the upper flow line and the boundary of the seepage zone in unconfined flow. Therefore, in such analyses seepage occurs below the phreatic surface but not above it.

The implicit basis of this assumption is that when the pore water pressure reaches zero (as occurs at the phreatic surface) the water in the void space of the soil pores immediately drains under gravity forces. If this happens air will enter the voids to take the place of much of the water, and the soil becomes unsaturated. It is well established that the permeability of a soil reduces dramatically once the soil becomes less than fully saturated (Fredlund *et al*., 1994), as shown in Figure 5. Therefore, if the soil is unsaturated above the phreatic surface, that surface becomes the effective boundary of the zone of water flow, since the permeability above the phreatic surface is very low.

However, the void spaces in fine grained soils, especially clays, are so minute that the soil is likely to remain fully saturated for metres or tens of metres above the phreatic surface, under the action of capillary forces. In general, a soil will begin to desaturate only when the pore water pressure u falls below the air entry pressure u_a (u_a is less than atmospheric and so has a negative pore water pressure (suction) when expressed as ‘gauge pressure’ relative to atmospheric). There are several formulae to relate the value of u_a to particle size, including the empirical formula given by Terzaghi *et al*. (1996)

$$-\frac{0.05\gamma_w}{eD_{10}} < u_a < -\frac{0.01\gamma_w}{eD_{10}} \qquad (6)$$

where e is the void ratio (the volumetric ratio of total voids to total solids in a soil), D_{10} is the 10% particle size in mm and γ_w is the unit weight of water in kN/m^3. The height of the capillary saturated zone is approximately equal to u_a/γ_w . Figure 6 shows the range of u_a/γ_w evaluated from equation 6 for e = 0.4–0.6 with permeability estimated from D_{10} using Hazen’s rule ($K = 0.01D_{10}^2$; K is in m/s and D_{10} is in mm: Hazen, 1892).

While the correlation between D_{10} and permeability is only very approximate for fine-grained soils, Figure 6 illustrates that the height of the capillary saturated zone may be several metres in low permeability soils such as clays and silts. This is consistent with published data such as Das (2002) which gives a range of 7 m to 23 m for the capillary height in clays, while Lancellotta (1995) quotes at least 10 m. Therefore in fine-grained soils the phreatic surface should not be expected to be the upper boundary of the seepage zone. Water can seep through the fully saturated zone both above the phreatic surface and below it, governed by the same principles, namely Darcy’s law.

It is interesting that relatively little of the geotechnical literature dealing with seepage problems addresses the potential effect of seepage through the capillary saturated zone. Hall (1955) identified from physical modelling in a sand tank that seepage through the capillary zone increased the total flow to a well in fine sand. Chapman (1960) used electrical analogue modelling and reached a similar conclusion for flow to a drainage slot. Decades later Powrie and Preene (1994) investigated the role of the capillary saturated zone in flow to vacuum wells used for construction dewatering. These studies all concluded that in fine-grained soils the upper flow boundary should be taken, not as the phreatic surface, but rather the top of the capillary saturated zone (this had previously been proposed by Childs (1945)). We shall see shortly that this had been demonstrated earlier by Dutch engineers using seepage tank experiments in 1936 (Figures 11 and 12).

Flow Net Theory for Steady State Seepage in Two Dimensions

Darcy's law describes flow in one dimension only, and must be combined with the law of continuity of mass (of groundwater flow) to address more complex problems. Mathematical solutions to two-dimensional groundwater flow problems were developed in the late 19th century by the Austrian hydraulic engineer Philipp Forcheimer (Forcheimer, 1886). He recognised that two-dimensional steady-state groundwater flow was governed by the Laplace equation, which expressed in two dimensions is:

$$\frac{\partial^2 h}{\partial^2 x} + \frac{\partial^2 h}{\partial^2 y} = 0 \qquad (7)$$

where *x*, *y* are the two-dimensional coordinates and *h* is total head.

The Laplace equation also describes conduction phenomena including the flow of heat and electricity. This showed the potential for mathematical solutions from other fields to be applied to groundwater problems, and to use analogue methods (such as models using the conduction of electricity) to model groundwater systems. In the 21st century world where many engineers and analysts have ready access to numerical modelling tools, it is easy to forget that analogue models, especially electrical resistance or resistance-capacitance models were used until relatively recently to analyse complex groundwater problems, both in academia and industry (Rushton and Redshaw, 1979). As late as the 1980s an electrical analogue model was used to design the groundwater control system for the construction of a nuclear power station at Sizewell B in the UK (Knight *et al.*, 1996).

Forcheimer is generally credited with the development of the 'flow net' as a graphical solution of the Laplace equation to analyse two-dimensional groundwater flow problems. A flow net is a network of 'flow lines' and 'equipotentials' that, when correctly developed, provide a graphical solution to a steady state groundwater flow regime in two dimensions (Figure 7).

- Flow lines represent the paths along which water can flow within a two-dimensional cross section.
- Equipotentials are lines of equal total head.

Consideration of Darcy's law (equation 3) shows that groundwater flows from higher total head to lower total head. Because each equipotential represents constant total head there can be no flow along an equipotential. Therefore (under isotropic conditions) each flow line and equipotential must intersect at right angles. Practical guidance on constructing flow nets is given in Cedergen (1989), including their application under anisotropic permeability conditions.

The earliest comprehensive paper on seepage in civil engineering situations that links to modern geotechnical practice is that of Casagrande (1937), which uses the phreatic assumption that the phreatic surface is the upper boundary of the seepage zone, as indicated in Figure 7. Casagrande describes and discusses various ways of establishing flow nets including seepage tank experiments, electrical analogues, and strongly recommends graphical hand sketching of flow nets.

Casagrande also makes the comment that one of the disadvantages of the electrical analogue method is "*that it does not permit the direct determination of the line of seepage (phreatic surface) for those problems in which the upper surface is not a fixed boundary*". We shall see later in this paper that this supposed disadvantage was based on an erroneous understanding of the upper limit of the seepage zone in a clay embankment. In clays with deep capillary saturated zones the limit is the ground surface, not the phreatic surface.

Terzaghi (1943) closely follows Casagrande in his treatment of seepage, as shown in Figure 8, and in his text book (page 242) attributes this drawing to Casagrande. These two papers, by the most influential figures in this field at that time, appear to have established the belief that the phreatic surface, at least with respect to seepage through embankments, defines the upper limit of the seepage zone. Sherard *et al* (1963 pg 276) reproduce this figure in their section on seepage through embankments. Actually a cursory examination of this flow net shows it is not a good example. At the start of the flow net near point 'a' the vertical **distances** between equipotential lines at their intersection with the phreatic surface are quite large, while at the central portion of the phreatic surface they are very small (the commonly applied rules of flow net construction require that the 'head drops' between neighbouring equipotentials should be the same throughout the flow net (Cedergren, 1989)).

Casagrande also proposed an empirical construction for establishing the phreatic surface for seepage through homogeneous embankments. This is shown in Figure 9, and found also in Taylor (1948) and Craig (1992). Casagrande (1937) gives relationships between X and Z and other dimensions of the cross section.

Specialty books devoted entirely to geotechnical seepage issues, such as Cedergren (1989), and Harr (1962), also assume that the phreatic surface is the upper limit of the seepage zone. One of the few text books to seriously address the pore pressure state above the phreatic surface in earth embankments appears to be that of Scott (1965). Scott describes the phenomena of capillarity and points out that water will rise in the voids of a fine grained soil for some distance above the atmospheric pressure level. He recognises there will be a "capillary fringe" above the phreatic surface, the thickness of which depends largely on the particle size of the soil. Scott states that some flow will occur through this capillary fringe, but does not suggest a way in which this can be taken account of in establishing the flow net. He mentions that it may account for an increase in flow of 10% or possibly higher. In modern soil mechanics texts, Powrie (2013) is one of the few to highlight that in low permeability soils the phreatic surface may not be the upper boundary of groundwater flow; however this is an exception rather than a rule in the literature used by geotechnical engineers.

Mention should be made at this stage of the development of mathematical solutions to groundwater flow within the field of hydrogeology, a discipline we could describe as groundwater mechanics. It is interesting that this form of groundwater mechanics appears to have developed in parallel with soil mechanics. but for a different reason, where the methods were used for problems concerned primarily with natural seepage and the management and use of groundwater as a resource. Just as soil mechanics forms the theoretical basis of geotechnical engineering, groundwater mechanics forms the theoretical component of hydrogeology. Both recognise the early work by Darcy, Dupuit (1863) and Forchiemer, and have developed additional analytical solutions relevant to their particular concerns. Soil mechanics is often concerned with engineering structures and, for simplicity in analyses, commonly simplifies 3-dimensional problems into 2-dimensonal vertical cross sections of such structures through which, or under which, water is seeping. In these two-dimensional analyses, much use is made of flow nets with boundaries of limited extent, as in Figures 7 to 9 above. Groundwater mechanics, on the other hand, is concerned with natural seepage and frequently examines three-dimensional problems over very large areas, as for example when investigating the behaviour of aquifers used for water supply, and the influence on them of water extraction from wells, or the dewatering of deep excavations. The text books by Harr (1962) and Strack (1989) show clearly the different interests and approaches of the two disciplines.

The discipline of hydrogeology addresses many situations but its primary focus is relatively permeable coarse grained water-bearing layers (aquifers) which typically can only sustain gentle hydraulic gradients.

Books addressing the practical study of groundwater issues such as *Groundwater by* Freeze and Cherry (1979) and *Groundwater Mechanics* by Strack (1989), contain a range of analytical solutions to various groundwater seepage situations, many of which are based on Dupuit's assumption that the equipotential lines are essentially vertical and the hydraulic gradient is the same as the slope of the phreatic surface, which is assumed to be the upper boundary of the seepage zone. The solutions therefore are strictly applicable only to coarse grained soils. This discipline also appears to have helped contribute to the common belief that the phreatic surface defines the upper limit of the seepage zone. In coarse-grained aquifers this phreatic assumption is probably reasonable, but, as discussed earlier, is not appropriate for seepage through and below water retaining structures founded on fine-grained low permeability strata.

Numerical Solutions to Groundwater Seepage Problems

Numerical modelling tools are ubiquitous in modern geotechnical engineering, and seepage problems are often analysed in this manner. When applied to 2-dimensional problems these packages can be viewed as a method of determining flow nets. Early programmes such as GeoFlow required the user to make an estimate of the position of the phreatic surface, and create a mesh of elements below the surface. The software then determined the flow net. The conditions along the "estimated" phreatic surface were then examined and adjustments made to meet the required conditions. As the phreatic surface was moved up or down, the whole mesh was "stretched" or "shrunk" in proportion to maintain the same basic shape. Newer and more advanced commercial tools, such as Seep/W, use a very different approach and for unconfined flow can include flow above the phreatic surface, controlled by the air entry and permeability characteristics of the soil under negative pore water pressures (as illustrated by Figure 5). This is an appropriate assumption since many civil engineering seepage studies involve clay, especially water retaining embankments made of compacted clay of low permeability.

Figure 10 illustrates the flow net determined by Seep/W for a compacted clay earth dam. Once the analysis is carried out, the software produces the solution for the equipotential lines and the phreatic surface, as shown in Fig. 10(a). The user can then add the flow lines to complete the flow net, as shown in Fig. 10(b). The phreatic surface (where pore water pressure is zero) is no longer coincident with a flow line; instead it is independent of them and may cross over them; equally the phreatic surface need not cross equipotentials at right angles. This is consistent with the earlier discussion that groundwater flow is controlled by total head, and not pore water pressure alone.

Physical Modelling of Groundwater Seepage

A very significant paper on seepage through an embankment is contained in the first international conference on soil mechanics held in Harvard University in 1936. The paper, by Mourik Broekman and Keverling Buisman (1936), describes flow patterns through a model embankment as determined by experiments using a seepage tank. Figure 11 shows a photograph of the seepage tank and the dye tracers indicating flow lines. The material is apparently a very fine sand, so that there is a zone of full, or nearly full saturation, under capillary effects above the phreatic surface.

Figure 12 shows the experimentally derived flow lines and equipotentials (which form part of a flow net) based on the information from the seepage tank experiments. .In their paper, Mourik Broekman and Keverling Buisman make the following comment:

> *The results of these experiments show that the capillary water participates in the water movement and forms a continuous picture of flow with the rest of the flowing water, and also that the phreatic curve does not coincide with a curve of flow (flow line). In theoretical treatises it has often been assumed that these curves do coincide.*

It seems surprising that Casagrande does not refer to the findings of Broekman and Buisman in his 1937 paper. Casagrande was the leading organiser of that conference and Terzaghi the main speaker. On the one hand we can perhaps understand Casagrande not taking much notice of the technical content of papers presented at the conference because of the unexpectedly large number of papers submitted and the huge workload this gave him and his team in processing the papers and getting the proceedings printed. On the other hand, Holland's historic experience with water retaining structures and the large number of papers it had in the conference ought to have attracted the attention of the organisers. The USA had twenty three papers in the conference and Holland was close behind with sixteen. No other country had more than six papers.

Of special significance also is the tribute paid by Mourik Broekman to Keverling Buisman at the second conference of the ISSMFE held in Rotterdam in 1948. Keverling Buisman was a leading figure in the development of soil mechanics in Holland, but sadly died young in a Japanese internment camp in the Dutch East Indies (now Indonesia) in 1944. That tribute contains the following statement:

> *"Another important contribution was the theory, formulated in collaboration with others, about the flow phenomena of the continuous capillary groundwater which, as demonstrated by experiments, follows the same laws as the "phreatic" water. As far as we know, this is the first time that equilibrium computations by means of the so called Swedish method of assumed circular surface of sliding, were performed taking account of the positive and negative pore water pressure."*

Hand Sketching Unconfined Flow Nets

As described earlier, flow nets are essentially a graphical solution of the Laplace equation in two dimensions. It is easy to think in the world of readily available software packages that hand drawn flow nets have no value to the geotechnical professional. This is not the case as preparing a flow net requires the problem to be defined and understood on a fundamental level, and can give an analyst real insight into the nature of the groundwater flow regime.

In some ways, hand sketching unconfined flow nets becomes easier when the material is clay (compared to a free-draining sand), as the upper boundary of the seepage zone is no longer unknown; it can now be assumed to be the ground surface. The procedure for sketching the flow net is illustrated in Figure 13. The first step is to sketch the flow lines, and to do this we should first try to envisage the way water will flow within the space available. The next step is to put in some guide markers, as shown in Figure 13(a). In doing this the important criteria is that the more curved the flow paths the narrower will be the flow channel. This means the guide markers should be closer together where curvature is greatest. Using these markers the flow lines can be sketched in. The authors' view is that for most situations three flow lines creating four flow channels is appropriate. The three flow lines are shown in Figure 13(b).

The next step is to start from one end and sketch in the equipotential lines, trying to create "squares" as well as possible. A helpful guide in doing this is to imagine circles fitting into each square, as shown in Figure 13 (c). The final step is to determine the phreatic surface, as this becomes the reference line from which to determine pore pressures throughout the embankment. To do this, count the number of head drops (N_e) and then divide the total head loss by this number. This gives us the head loss (and the vertical intercepts) along .the phreatic surface between each equipotential line. By drawing a series of horizontal lines as shown in Figure 13d) we determine the points on each equipotential line that represent zero pore pressure.

Practical Examples

An Embankment Dam

Figure 14 shows two flow nets for a water retaining embankment dam. The flow net in Figure 14(a) shows the conventional portrayal using the phreatic assumption with the phreatic surface as the upper limit of the seepage zone. As already explained this is only true for a coarse-grained material. It is extremely unlikely that a dam of this nature would be built of a coarse material. The flow net in Figure 14(b) assumes that capillary forces allow the embankment to remain saturated above the phreatic surface. The complete cross section is available for water to seep through, in which case the phreatic surface intersects a number of flow lines

However, the actual position of the phreatic surface is only slightly changed, so adopting the upper flow net will not result in significant errors as far as the pore pressure below the phreatic surface is concerned. It will, however, result in an underestimate of the seepage flow rate. As the number of potential drops is the same for both flow nets, the increased seepage is given by the ratio of the number of flow channel. This is approximately 6/3.5 = 1.71, so that the true seepage rate will be about 70% higher than that obtained using the traditional portrayal of the flow net. This is a significant increase and is representative of conditions in a fine-grained low permeability soil, where the zone above the phreatic surface remains fully saturated. The percentage increase in flow rate would become steadily less with lower degrees of saturation.

Hillside Seepage

Although this paper focuses mainly on seepage through water retaining embankments, its findings have implications for a variety of seepage situations including seepage in hillsides. Figure 15 shows two possible flow nets compatible with the same water table. The first one shows the pattern regularly assumed to exist if a water table is known to have the position in Figure 15(a). This implies that there is a fixed source of water at some distance beyond line a-b. Figure 15(b) shows the seepage pattern if the hill is symmetrical about the line a-b and there is steady rainfall at the surface. The seepage pattern will be as shown but the position of the water table depends on the intensity of the rainfall compared to the rate at which water can seep through the hillside, that is the "capacity" of the soil to accept the rain falling at the surface.

The very different seepage patterns illustrated in Figure 15 show that knowledge of the phreatic surface is not adequate to define the pore pressure state in the hillside. More complete information is needed to establish a reliable flow net and to determine realistic distributions of pore water pressures. This can have important consequences with respect to estimates of the stability of hillsides.

Leakage From a Pond

Figure 16 shows possible patterns for seepage coming from a leaking pond. Figure 16(a) is taken from Strack (1989), who envisages the leaking water trickling down through the ground to eventually be absorbed into the flow in the underlying aquifer. Although not explicitly stated this implies that the material is relatively coarse and water "trickles" down vertically through essentially unsaturated material. If the material is fine grained, especially as fine grained as a typical clay, then the seepage pattern will be quite different, as indicated in Figure 16(b). In this case all of the soil through which flow is taking place is essentially fully saturated, and the base of the pond, along with the natural ground surface, becomes an essential boundary of the seepage zone. The flow net in Figure 16(b) has been obtained using the programme Seep/W, and the rate of leakage from the pond adjusted to produce a water table the same as that in the Strack figure.

Drawdown of Water Level Beside an Embankment Dam
Figure 17 illustrates the transient seepage pattern in an embankment following drawdown of the reservoir water level. Figure 17(a) is redrawn from Cedergren (1989) and shows the phreatic surface steadily falling as water drains out of the embankment. The soil below the phreatic surface is fully saturated and that above it unsaturated. The soil parameters governing this process are the permeability and the porosity. These respectively govern the rate at which water flows and the volume of water to flow, leaving the soil unsaturated. This process therefore only applies to coarse-grained soils out of which the majority of pore water will flow under its self weight (under gravity forces). No change in the volume of the material occurs in this process.

Figure 17(b) shows the situation for a clay embankment. In this case the soil remains fully saturated but undergoes consolidation and volume change as the effective stress steadily increases, as pore water pressures reduce. The soil parameters that govern this process are the permeability and the compressibility, or in their combined form the coefficient of consolidation. The process here is essentially the same as that in Terzaghi 1-dimensional consolidation, although the process is no longer one dimensional.

It is useful to note that in the drainage for coarse-grained soils in Figure 17(a), the assumption is made that there is no volume change in the soil; thus the volume of water entering any particular element is the same as that leaving it. This means that seepage is still governed by the Laplace equation, and it is possible to draw flow nets as done by Cedergren and shown in Figure 17(a). This is no longer the case with the clay embankment. Volume change is occurring continuously and flow is no longer governed by the Laplace equation. A closed form flow net can no longer be drawn as some flow lines originate within the soil mass rather than at a specific boundary.

Conclusion
An almost universal seepage assumption adopted in the early years of soil mechanics development has been shown to be invalid for fine-grained soils such as clay. This is that the phreatic surface (the line along which pore water pressure is zero) is a dividing line below which there is seepage and above which there is none. This assumption is only true when the material is coarse-grained and the majority of the pore water above the phreatic surface readily flows out of it under gravity forces, allowing air to enter, leaving the void space in the soil above the phreatic surface to be essentially unsaturated. In clays, the smaller pore sizes mean that capillary forces hold water in the soil above the phreatic surface; soil remains fully saturated and seepage continues both above and below the phreatic surface in accordance with the Laplace equation. The phreatic surface does not constitute a discontinuity in the seepage pattern.

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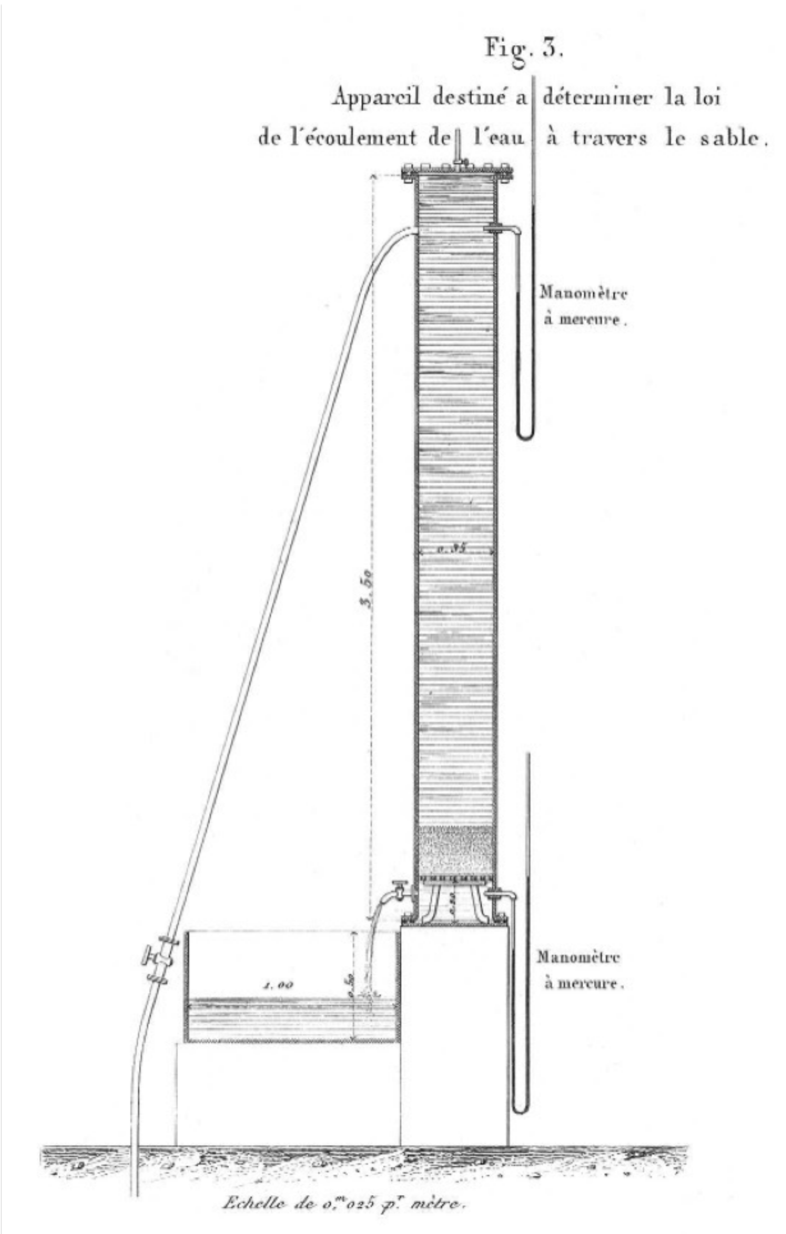


Figure 1 Darcy's original sand column apparatus (Darcy 1856, Plate 24, Fig.3)

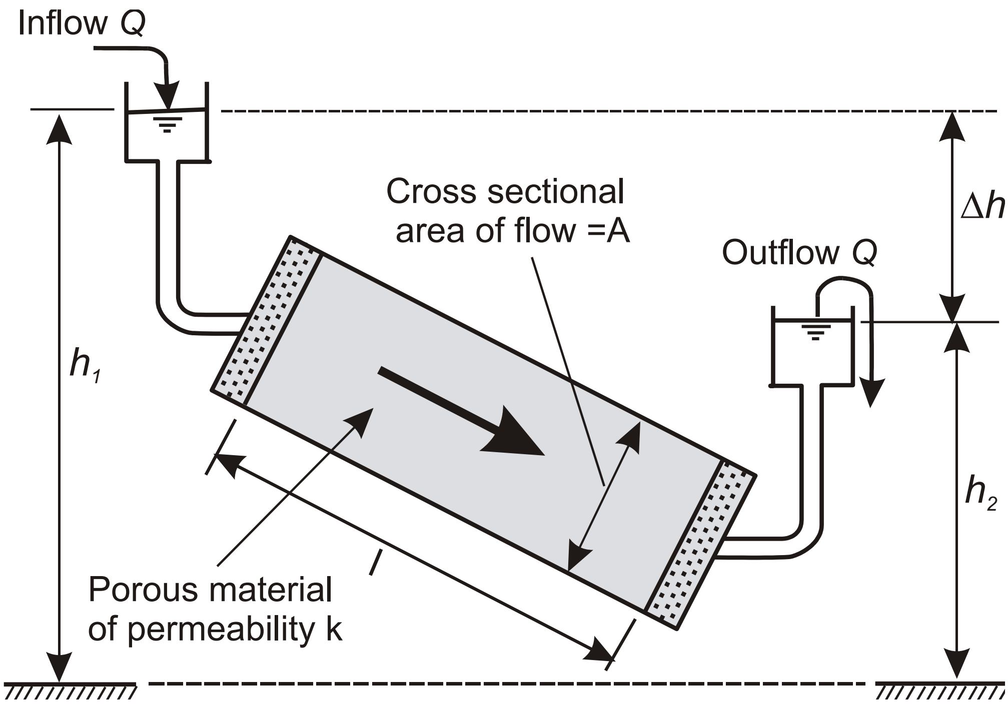


Figure 2 Darcy's experiment (conceptual)

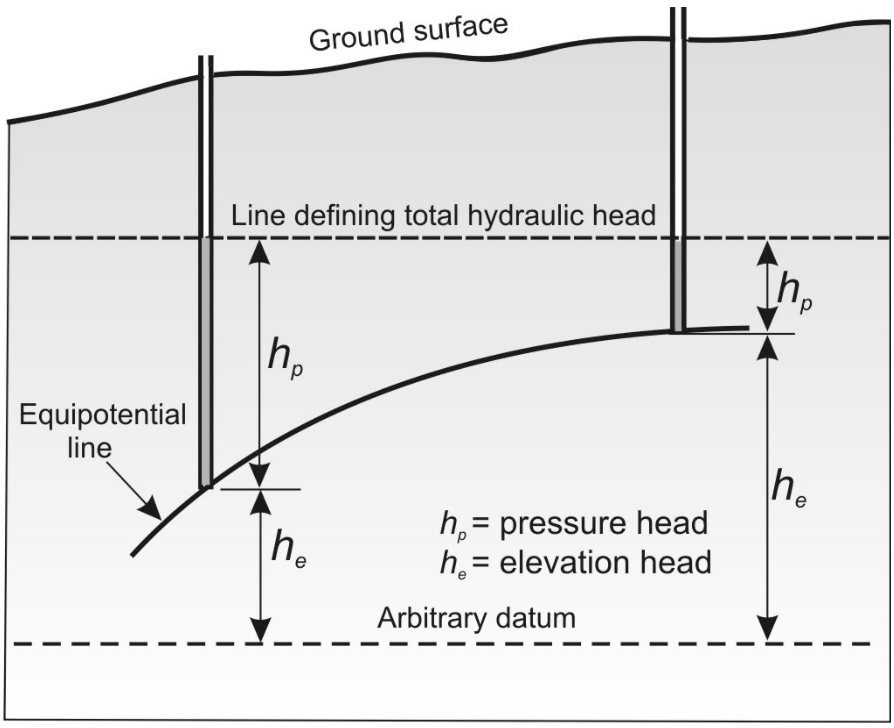
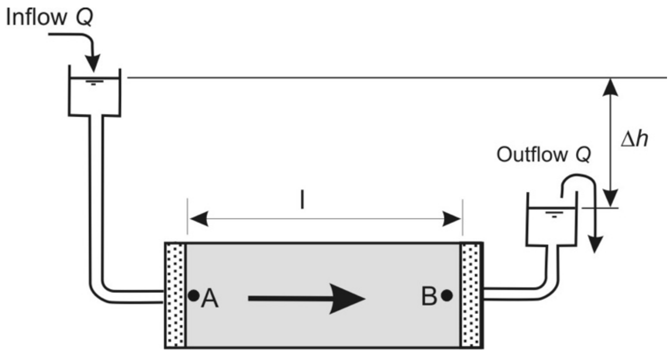
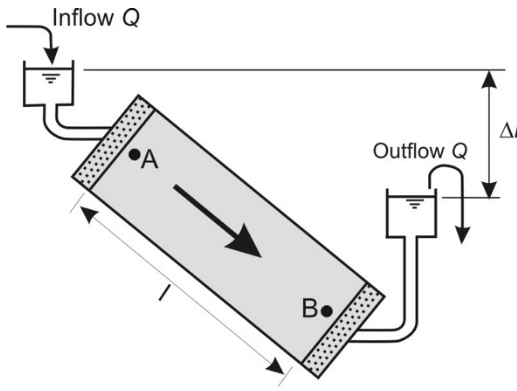


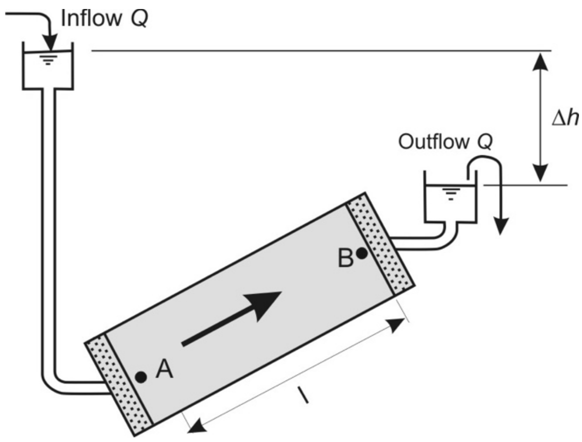
Figure 3 Definition of total hydraulic head



a) *There is no change in elevation between A and B – groundwater is flowing from high pressure to low pressure areas*



b) *Due to the change in elevation, the pressure at A is lower than at B, and groundwater flows from low pressure to high pressure areas*



c) *The excess head Δh drives groundwater flow from A to B and means that water can flow 'uphill' along upward physical gradients*

Figure 4 Groundwater flow driven by differences in total hydraulic head

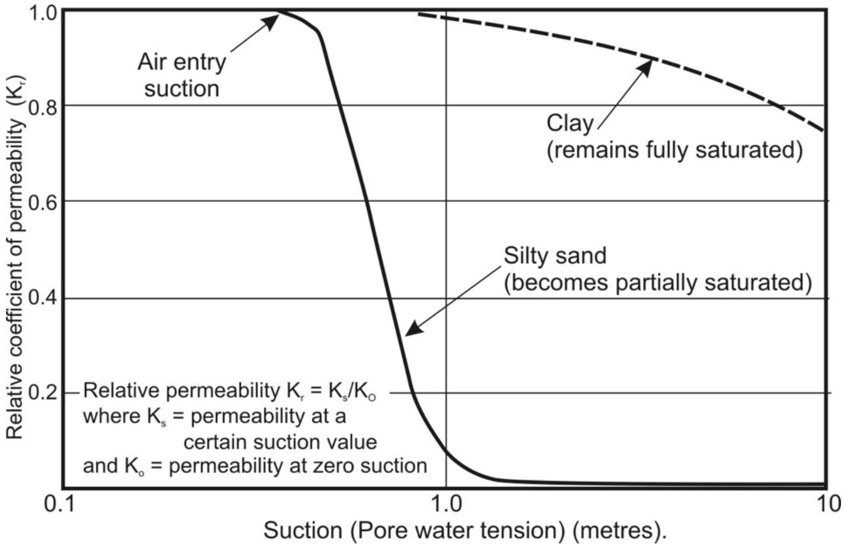
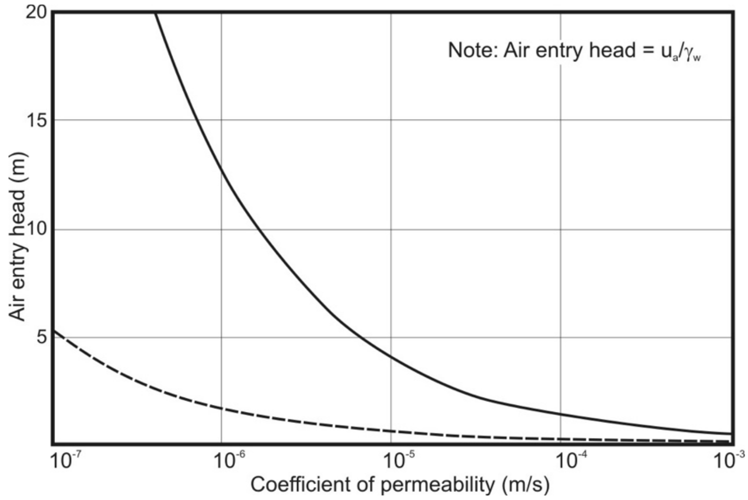


Figure 5 Example relationship between permeability of a soil and pore water pressure (for a for a silty sand, (based on Fredlund et al., 1994) and a clay (adapted from Mitchell and Soga, 2005)



Air entry head u_a/γ_w is evaluated from equation 6 for e = 0.4–0.6 with permeability estimated from D_{10} using Hazen's rule (K = $0.01D_{10}^2$; K is in m/s and D_{10} is in mm: Hazen, 1892)

Figure 6 Example of variation of air entry head with permeability (based on Powrie and Preene, 1994).

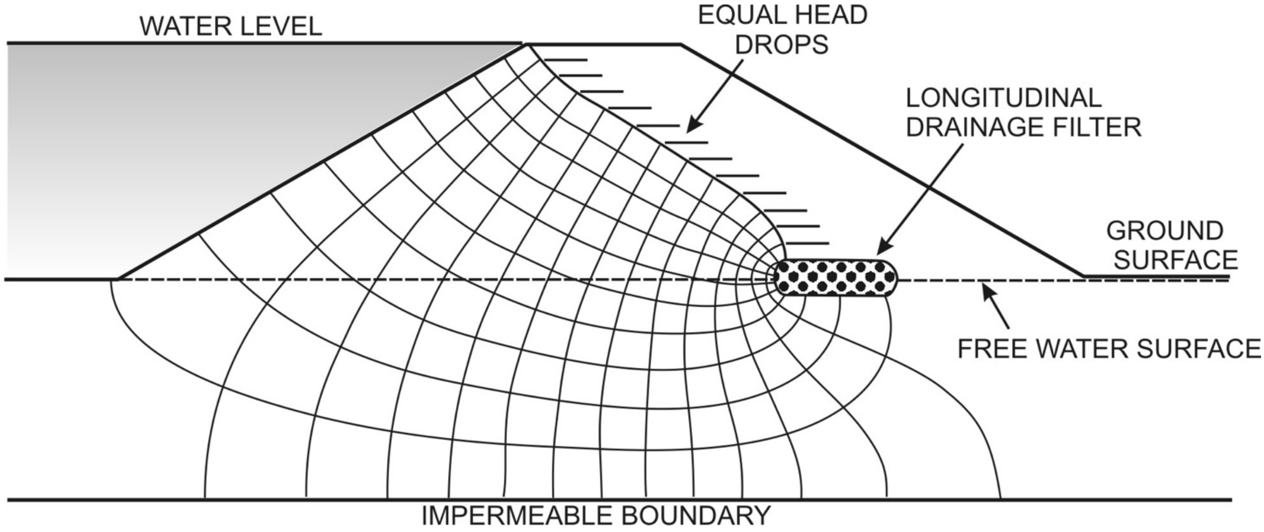


Figure 7 Flow net for seepage through a homogeneous earth dam having an intercepting drainage layer, according to Casagrande (1937)

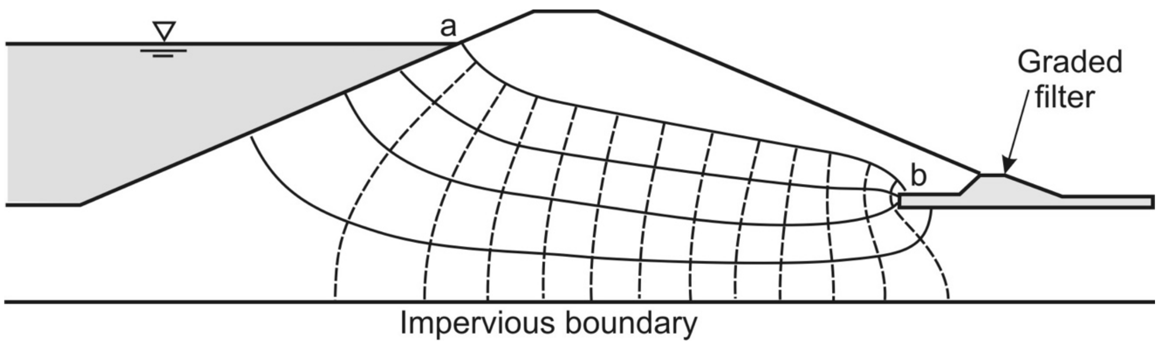


Figure 8 Flow net in an earth embankment, after Terzaghi (1943)

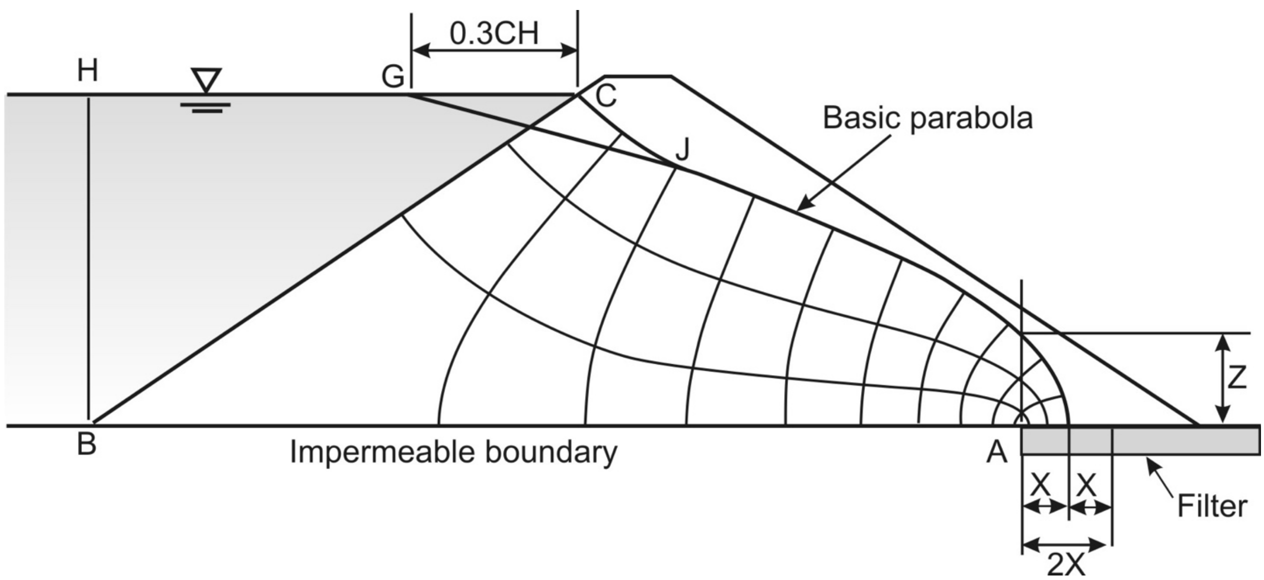
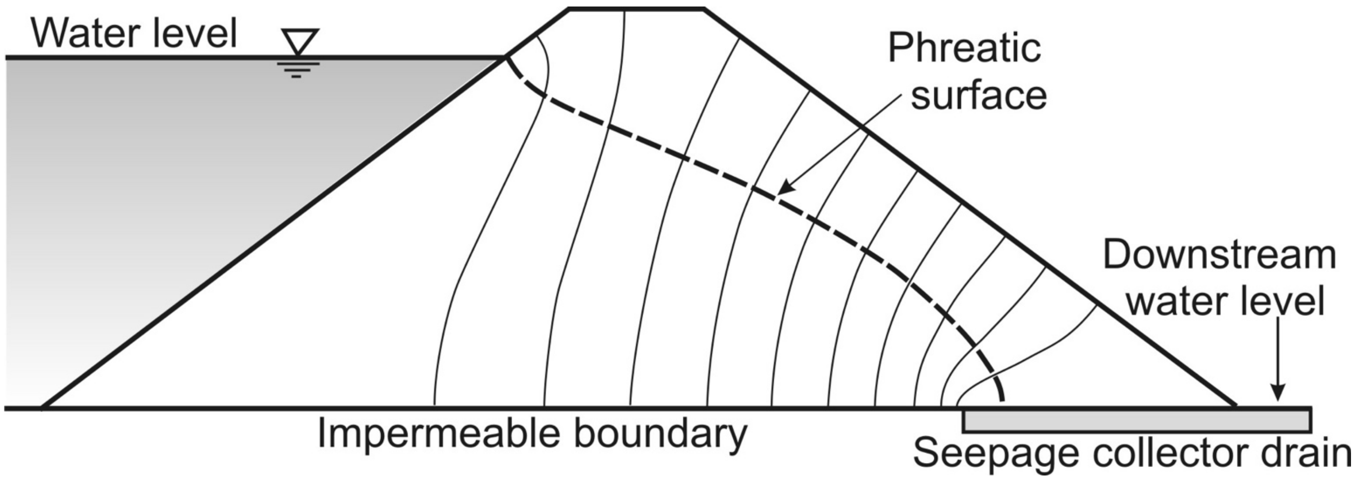
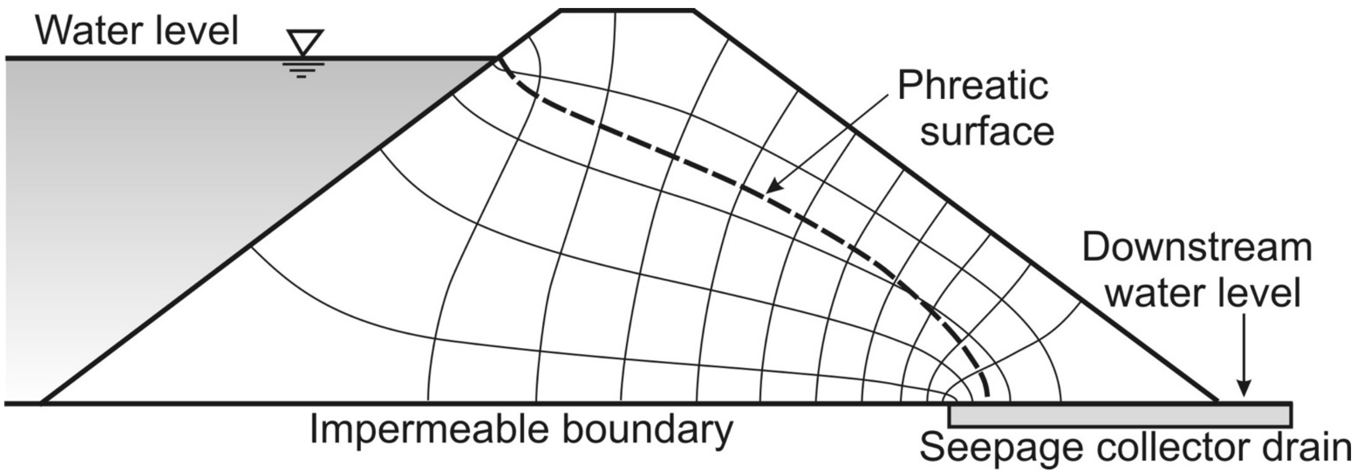


Figure-9 Casagrande construction for determining the phreatic surface in embankment dams



a) The equipotential lines and the phreatic surface as produced by the computer programme



b) The complete flow net after the flow lines have been inserted by the programme user

Figure 10 Flow net determination using the software package Seep/W

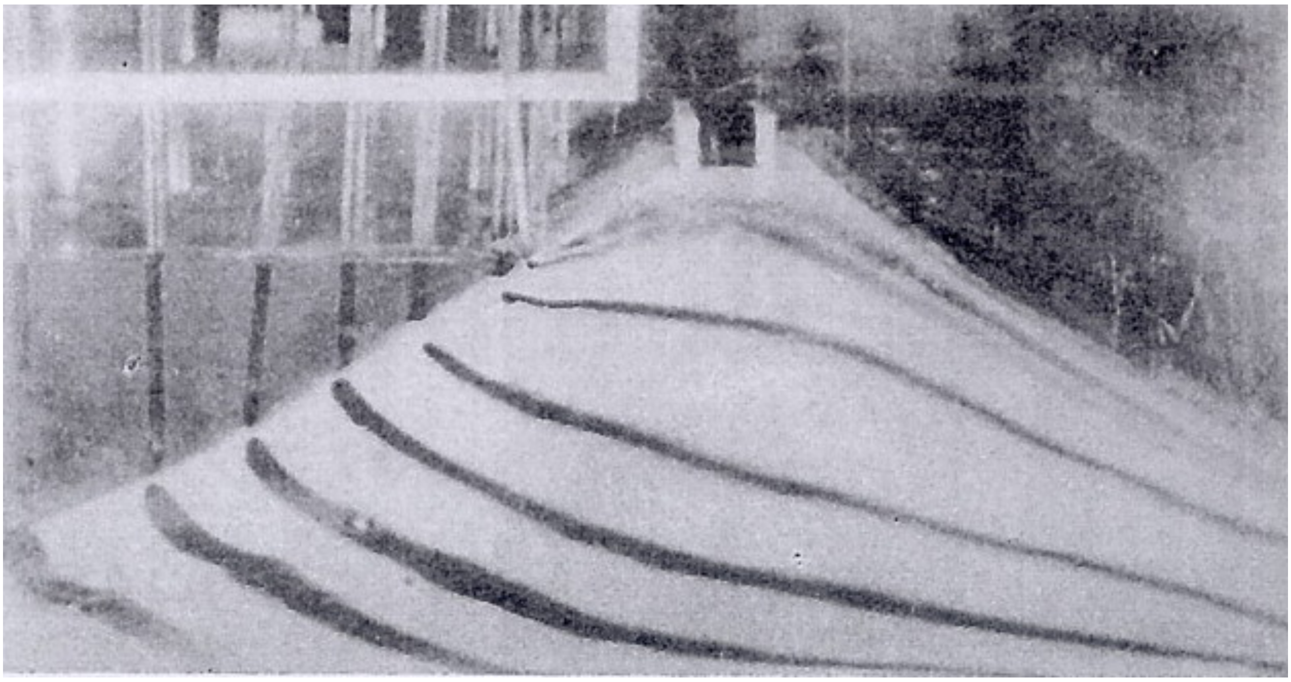


Figure 11 Seepage pattern in a model study in a seepage tank (after Mourik Broekman and Keverling Buisman, 1936)

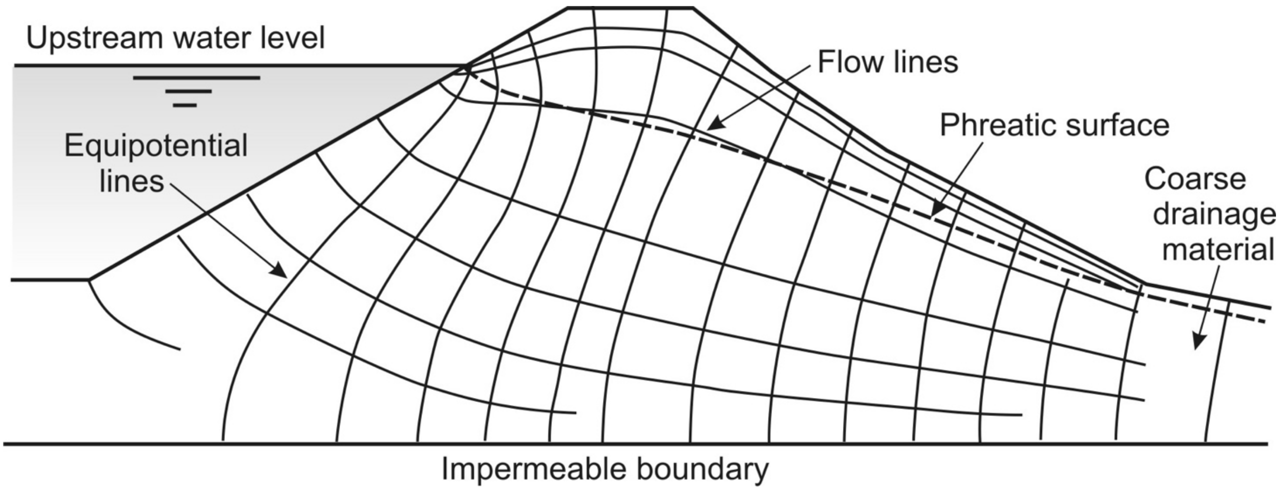
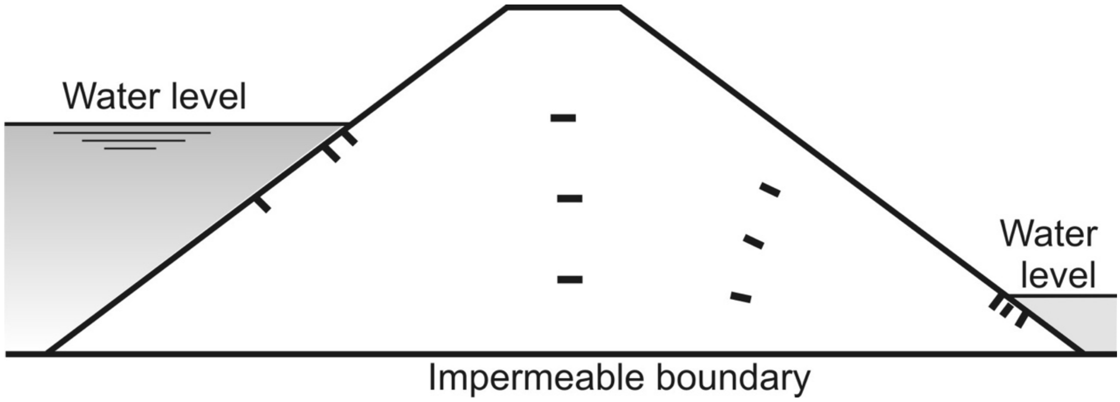
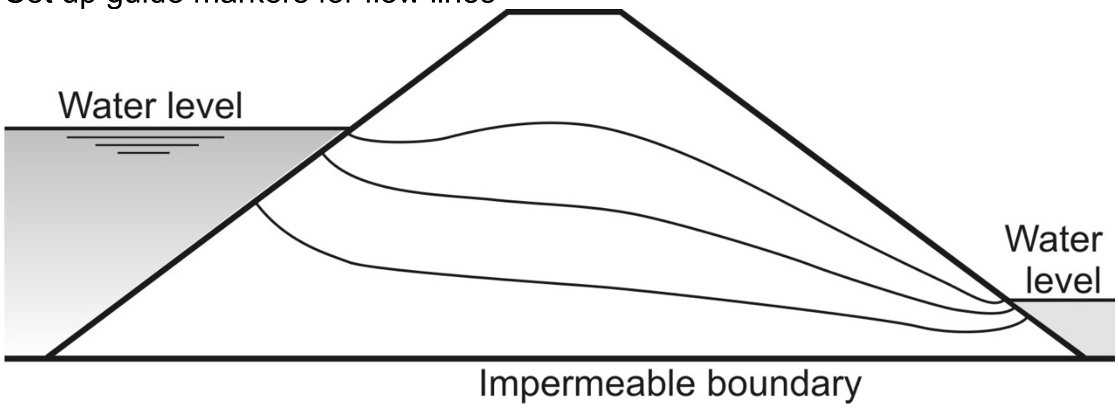


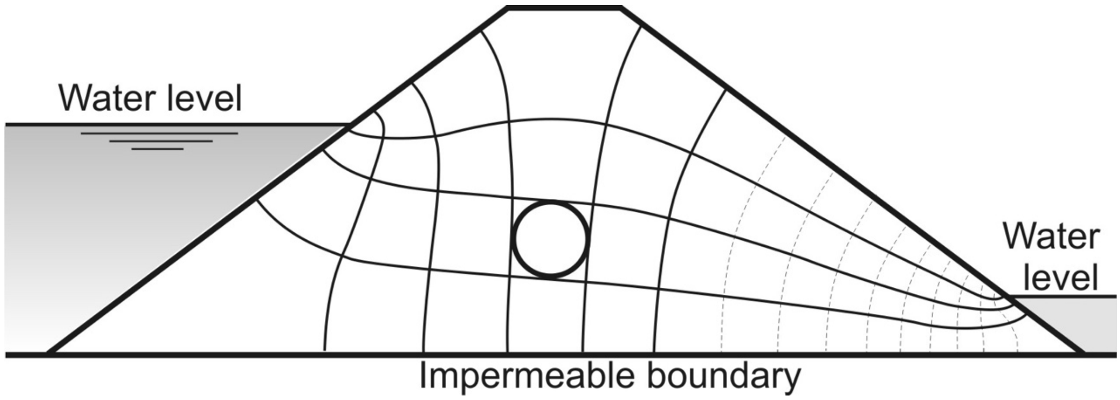
Figure 12 Flow net as determined from seepage tank studies (after Mourik Broekman and Keverling Buisman, 1936)



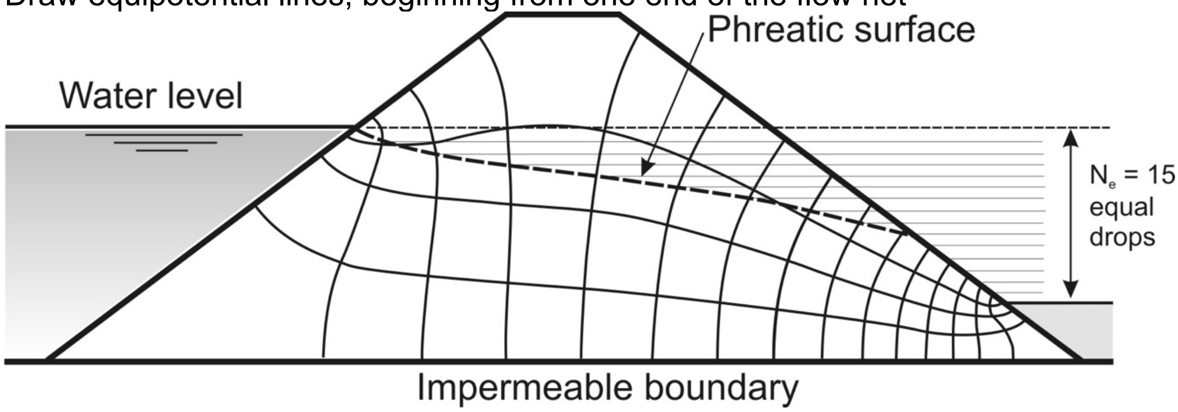
a) Set up guide markers for flow lines



b) Draw the flow lines

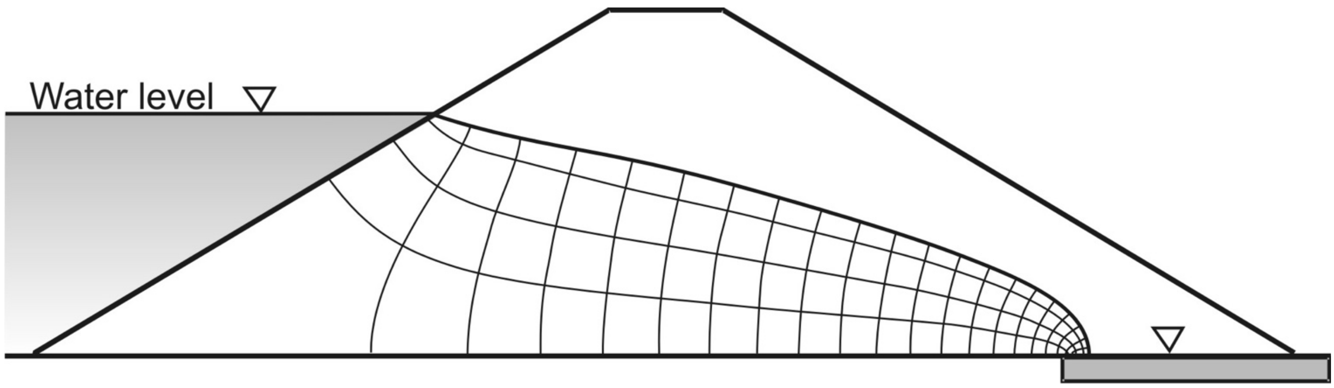


c) Draw equipotential lines, beginning from one end of the flow net

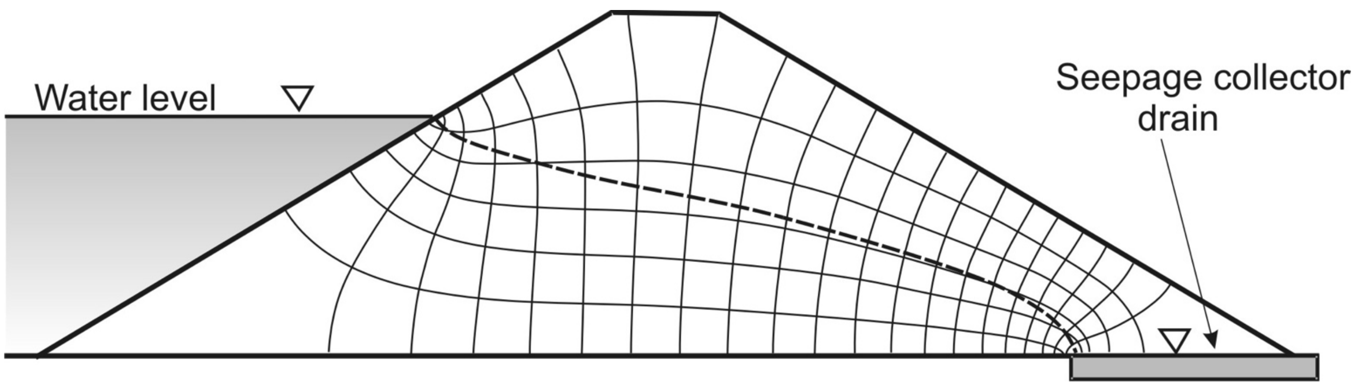


d) Divide the head loss by a number of potential drops (N_e = 15) then draw horizontal lines at each level to intersect the relevant equipotential. The intersection points define the phreatic surface

Figure 13 Procedure for sketching the flow net in a clay embankment

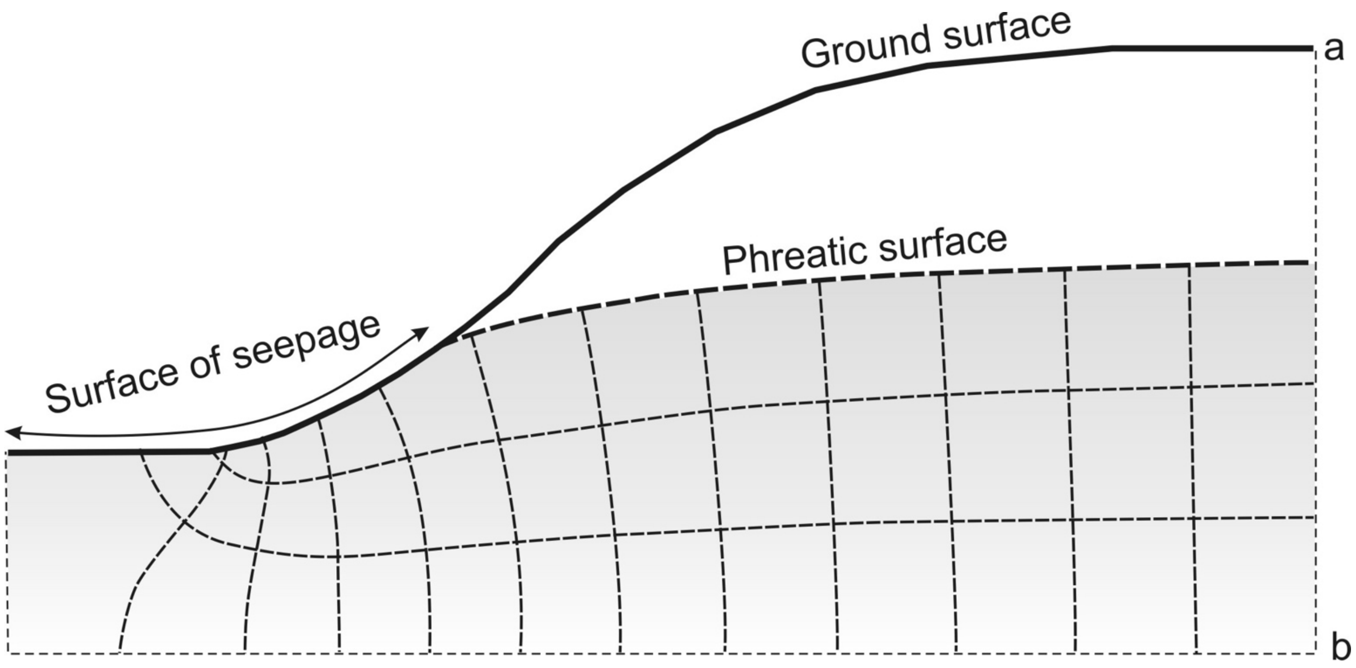


a) Seepage through a coarse material (gravel or clean sand)

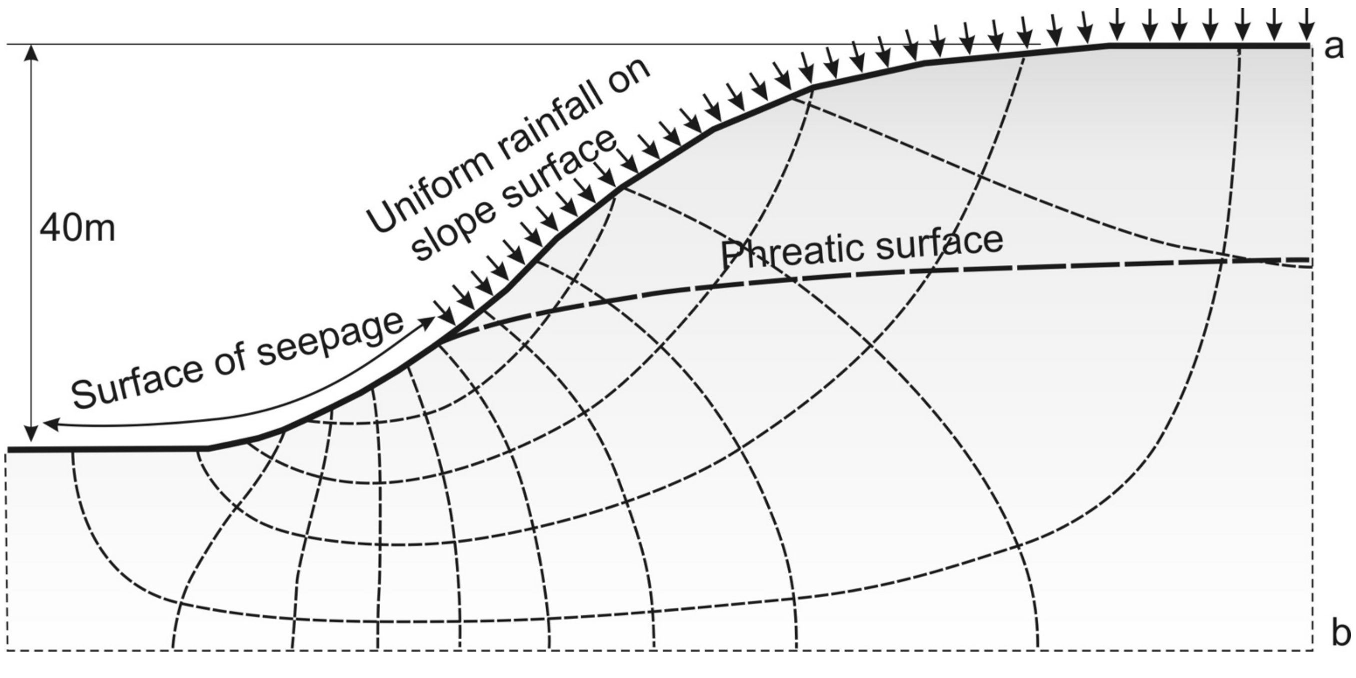


b) Seepage through a fine-grained material (clay)

Figure 14 Flow nets for seepage through a homogeneous embankment (after Wesley, 2010)

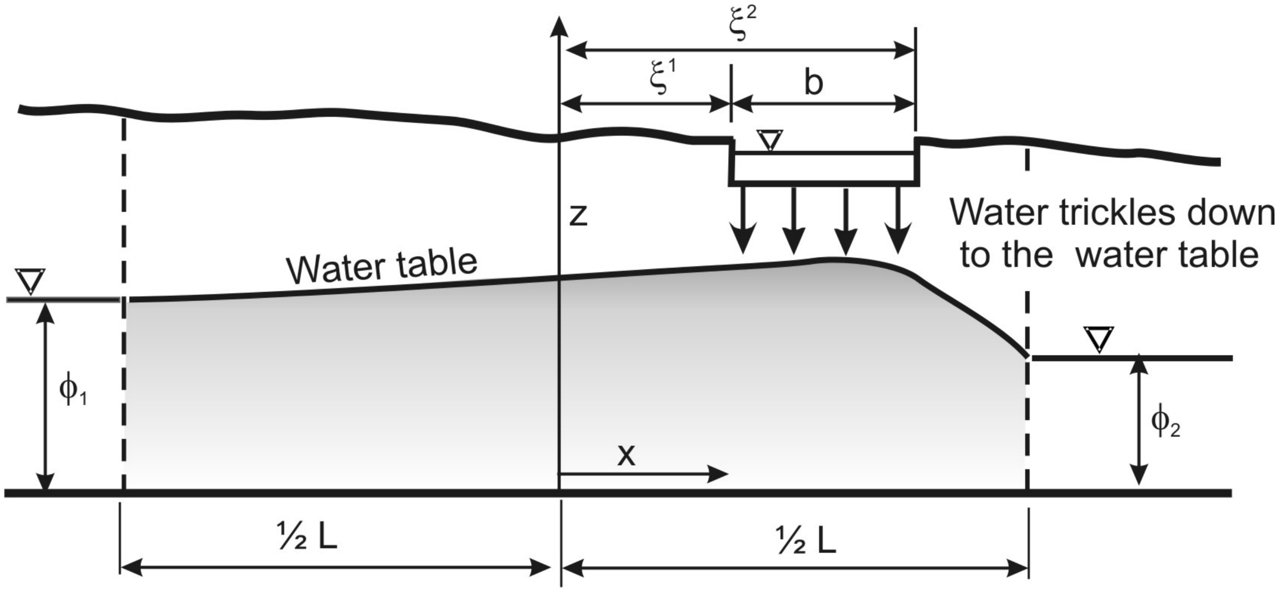


a) Conventional flow net corresponding to a known phreatic surface

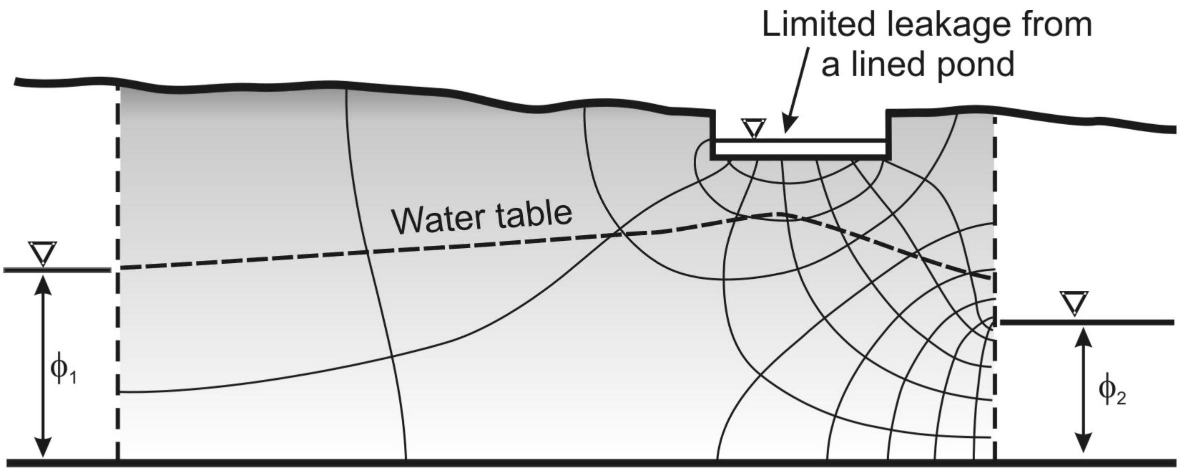


b) Seepage pattern for continuous rainfall of constant intensity

Figure 15 Seepage patterns compatible with the same water table

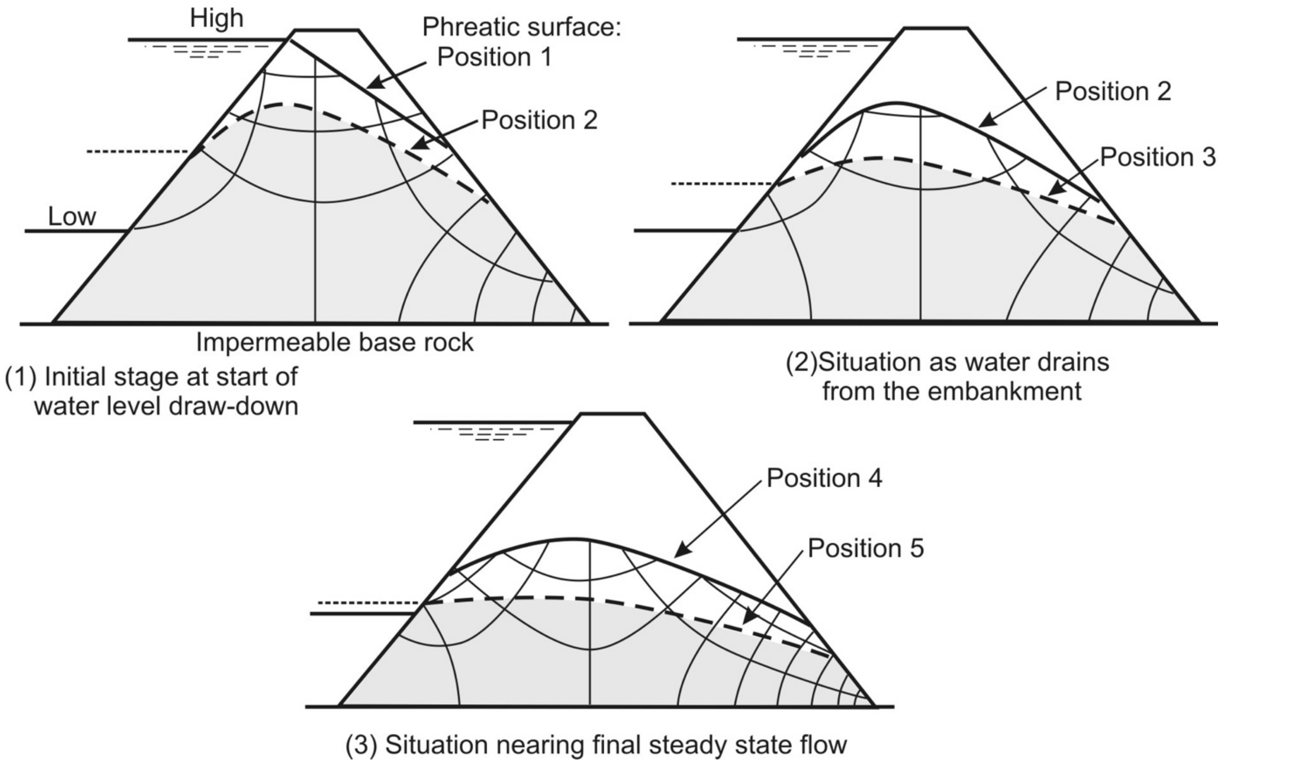


a) Seepage from a pond, after Strack (1989) for a coarse material

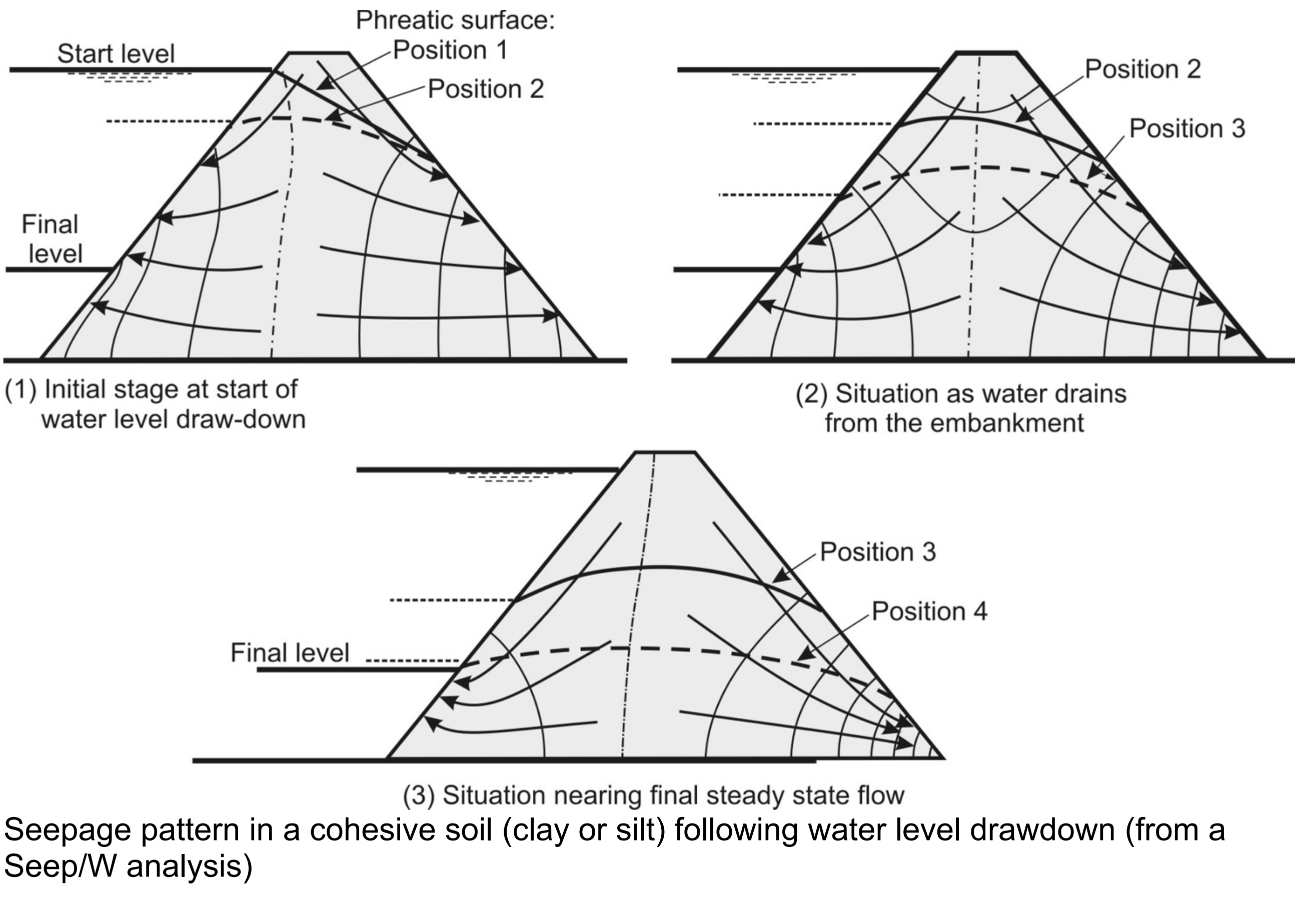


b) Limited seepage from a lined pond, through clay

Figure 16 Seepage pattern caused by leakage from a pond. (after Wesley, 2014)



a) Flow nets in a free draining material following water level drawdown (from Cedergren, 1989)



b) Seepage pattern in a cohesive soil (clay or silt) following water level drawdown (from a Seep/W analysis)

Figure 17 Seepage patterns following drawdown of the retained water level